Patterns and Freakness

How do you measure the freakness of a bridge hand? To most bridge players, the question is purely subjective, and you would get answers anywhere from “flat” or “square” for balanced hands to “wild and crazy” for fluke hands — the latter of which makes me wonder why Steve Martin never became a bridge player.

Years ago (I can’t believe it’s nearly 20) I worked out a method for this. The basic premise was that the flattest pattern (4-3-3-3) is given a “freakness” of zero (that surely makes sense) and all the other patterns gain points according to this rule:

Add 1 point for each card *over four* or *under three* in each suit.

The rule seemed to work OK at first; for example, 4-4-3-2 would get 1 point (one suit under three cards) and 5-3-3-2 would get 2 points (one suit over four, and one suit under three). Unfortunately, the scheme also awarded 2 points for 4-4-4-1 (one suit with two cards under three), thereby equating a balanced hand to a hand with a singleton. This had to be fixed. After some more thought I came up with two adjustments, which apply to the hand as a whole:

Add 1 point if the hand contains a singleton, or
Add 2 points if the hand contains a void.

Note that the above are exclusive; you cannot add both. If a hand contains a void, it gets 2 extra points period, even if it also happens to have a singleton.

So what’s the reason for all this? The purpose is to be able to quantify the freakness of a bridge hand on a linear scale, much like point count is used to evaluate its strength. I have used this formula for a number of computer applications, such as my deal database and “computer bidding” programs. It facilitates decision making based on the shape of a hand. For example, a balanced hand is now simply defined as a hand with a maximum freakness of 2.

Applying the formula to the 39 generic hand patterns gives pleasing results. The patterns are ranked in an order that feels right. When two different shapes produce the same freakness number, it usually is difficult to judge one as more freakish than the other. For example, the patterns 4-4-4-1 and 5-4-2-2 both calculate to 3 points, and it’s a moot question which should be considered more unbalanced.

Freakness Table

Pattern	F	Pattern	F	Pattern	F
4-3-3-3	0	6-4-3-0	7	9-2-1-1	11
4-4-3-2	1	7-3-2-1	7	9-2-2-0	12
5-3-3-2	2	6-5-1-1	8	9-3-1-0	12
4-4-4-1	3	7-3-3-0	8	7-6-0-0	13
5-4-2-2	3	7-4-1-1	8	8-5-0-0	13
5-4-3-1	4	6-5-2-0	9	9-4-0-0	13
6-3-2-2	4	7-4-2-0	9	10-1-1-1	13
6-3-3-1	5	8-2-2-1	9	10-2-1-0	14
5-4-4-0	6	8-3-1-1	9	10-3-0-0	14
5-5-2-1	6	8-3-2-0	10	11-1-1-0	16
6-4-2-1	6	6-6-1-0	11	11-2-0-0	16
7-2-2-2	6	7-5-1-0	11	12-1-0-0	18
5-5-3-0	7	8-4-1-0	11	13-0-0-0	20

Another pleasing aspect of the formula is the 0-to-20 scale, i.e., nice round numbers. Note that the extreme patterns grow quickly from 14 to 20, and the formula yields no freakness of 15, 17 or 19. I thought about the possibility of graduating the high-end values to 20, 19, 18, 17, etc., but this would only complicate the formula for no apparent advantage. Like, when was the last time you were worried about 12-1-0-0 shape?

Deal Freakness

Once the formula for hand freakness is developed, it becomes an easy matter to quantify the freakness of an entire deal. The nature of a deal is determined by the composition of the four hands, so the obvious conclusion would be:

The freakness of a deal is equal to the sum of the freaknesses of each hand.

Hence, the freakness of a deal ranges from 0 to 80, although the vast majority lie on the low end. There also are more unattainable numbers. For example, next to 80, the highest possible deal freakness would be 76 because if any of the 13-0-0-0 patterns were changed to 12-1-0-0, a second hand must also change. But we won’t be too worried about deals like that.

A curious puzzle is, “What is the *lowest* freakness value that cannot be obtained for a deal?” My calculations show the impossible numbers to be 79, 78, 77, 75, 73, 71 and 69, so the last would be the answer. Hmm…‘69 was also the year of the moon landing *and* my wedding; some might say there’s a correlation.

Average Freakness

The next question that comes to mind is, “What is the *average freakness* of a bridge hand?” This is not easily answered. There is no simple way to calculate it (at least I couldn’t think of one) so I approached it by brute force — the old pickax method.

For each hand pattern, I calculated the total number of bridge hands it could produce. This is done with combinatorial arithmetic. The number of combinations of N items taken R at a time (often abbreviated NcR) is determined by this formula:

$$\frac{N!}{R!(N-R)!}$$

The notation N! (N factorial) means the product of all integers from 1 through N. For example, 6! = 720. Also noteworthy is that 0! is defined as 1.

For example, to determine the number of 4-3-3-3 hands (say, with four spades) there are 13c4 ways to choose the spades, and 13c3 ways to choose each of the other suits. Multiplying (13c4 × 13c3 × 13c3 × 13c3) provides the total. But wait! That’s only the hands with four spades, so we must multiply by *four* to get the total number of 4-3-3-3 hands. This last factor indicates the number of ways the generic pattern can be permuted among the four suits, which will either be 4, 12 or 24, depending on the like digits in the pattern.

The values of the combinatorials (13c0 to 13c13) are shown in the chart below. Note the equality of any two numbers that add to 13, which should be obvious from the formula. (I have eliminated the “13” to save space in the chart and table.)

c0	**c1**	**c2**	**c3**	**c4**	**c5**	**c6**
c13	**c12**	**c11**	**c10**	**c9**	**c8**	**c7**
1	13	78	286	715	1287	1716

A calculation for all of the generic hand patterns is shown in the table that follows. To verify its accuracy, consider that the total of the hands column should equal the total number of bridge hands, which is 52c13 or 635,013,559,600. It does. Check it if you wish — just pretend you are balancing Bill Gates’s checkbook.

All that remains is to multiply the hand total of each pattern by its freakness in the right column (the first one shows that I wasted my time calculating 4-3-3-3 hands), add the results and divide by 635,013,559,600. The final tallies are shown at the end.

Rounding to two decimal places, the average freakness of a bridge hand is 2.98, and the average freakness of a full deal is four times that, or 11.93.

Number of Hands by Patterns

Pattern	Calculation	Hands	F
4-3-3-3	$4 \times c4 \times (c3)^3$	66,905,856,160	0
4-4-3-2	$12 \times (c4)^2 \times c3 \times c2$	136,852,887,600	1
5-3-3-2	$12 \times c5 \times (c3)^2 \times c2$	98,534,079,072	2
4-4-4-1	$4 \times (c4)^3 \times c1$	19,007,345,500	3
5-4-2-2	$12 \times c5 \times c4 \times (c2)^2$	67,182,326,640	3
5-4-3-1	$24 \times c5 \times c4 \times c3 \times c1$	82,111,732,560	4
6-3-2-2	$12 \times c6 \times c3 \times (c2)^2$	35,830,574,208	4
6-3-3-1	$12 \times c6 \times (c3)^2 \times c1$	21,896,462,016	5
5-4-4-0	$12 \times c5 \times (c4)^2 \times c0$	7,895,358,900	6
5-5-2-1	$12 \times (c5)^2 \times c2 \times c1$	20,154,697,992	6
6-4-2-1	$24 \times c6 \times c4 \times c2 \times c1$	29,858,811,840	6
7-2-2-2	$4 \times c7 \times (c2)^3$	3,257,324,928	6
5-5-3-0	$12 \times (c5)^2 \times c3 \times c0$	5,684,658,408	7
6-4-3-0	$24 \times c6 \times c4 \times c3 \times c0$	8,421,716,160	7
7-3-2-1	$24 \times c7 \times c3 \times c2 \times c1$	11,943,524,736	7
6-5-1-1	$12 \times c6 \times c5 \times (c1)^2$	4,478,821,776	8
7-3-3-0	$12 \times c7 \times (c3)^2 \times c0$	1,684,343,232	8
7-4-1-1	$12 \times c7 \times c4 \times (c1)^2$	2,488,234,320	8
6-5-2-0	$24 \times c6 \times c5 \times c2 \times c0$	4,134,297,024	9
7-4-2-0	$24 \times c7 \times c4 \times c2 \times c0$	2,296,831,680	9
8-2-2-1	$12 \times c8 \times (c2)^2 \times c1$	1,221,496,848	9
8-3-1-1	$12 \times c8 \times c3 \times (c1)^2$	746,470,296	9

8-3-2-0	$24 \times c8 \times c3 \times c2 \times c0$	689,049,504	10
6-6-1-0	$12 \times (c6)^2 \times c1 \times c0$	459,366,336	11
7-5-1-0	$24 \times c7 \times c5 \times c1 \times c0$	689,049,504	11
8-4-1-0	$24 \times c8 \times c4 \times c1 \times c0$	287,103,960	11
9-2-1-1	$12 \times c9 \times c2 \times (c1)^2$	113,101,560	11
9-2-2-0	$12 \times c9 \times (c2)^2 \times c0$	52,200,720	12
9-3-1-0	$24 \times c9 \times c3 \times c1 \times c0$	63,800,880	12
7-6-0-0	$12 \times c7 \times c6 \times (c0)^2$	35,335,872	13
8-5-0-0	$12 \times c8 \times c5 \times (c0)^2$	19,876,428	13
9-4-0-0	$12 \times c9 \times c4 \times (c0)^2$	6,134,700	13
10-1-1-1	$4 \times c10 \times (c1)^3$	2,513,368	13
10-2-1-0	$24 \times c10 \times c2 \times c1 \times c0$	6,960,096	14
10-3-0-0	$12 \times c10 \times c3 \times (c0)^2$	981,552	14
11-1-1-0	$12 \times c11 \times (c1)^2 \times c0$	158,184	16
11-2-0-0	$12 \times c11 \times c2 \times (c0)^2$	73,008	16
12-1-0-0	$12 \times c12 \times c1 \times (c0)^2$	2,028	18
13-0-0-0	$4 \times c13 \times (c0)^3$	4	20

Summary	
Total number of hands (above)	635,013,559,600
Total hands × freakness points	1,894,153,566,372
Average hand freakness	2.9828553071609
Average deal freakness	11.9314212286436